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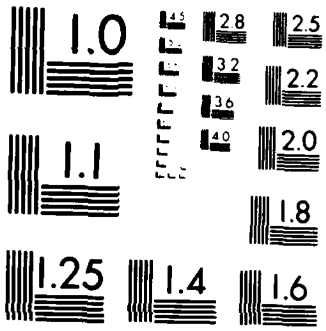

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Betatron-Synchrotron Detrapping in a Tapered Wiggler Free Electron Laser

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Plasma Physics Division

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Betatron-synchrotron resonance detrapping is shown to take place when the wiggler magnetic field amplitude is tapered. This resonance exists even if the radiation wavefronts are not curved and is only dependent on the transverse gradient of the tapered wiggler field.

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BETATRON-SYNCHROTRON DETRAPPING IN A TAPERED WIGGLER FREE ELECTRON LASER

It has been pointed out by M. Rosenbluth[1] that electron detrapping in the ponderomotive wave can occur if a resonance between the electron's synchrotron and betatron oscillations exist. Synchrotron oscillations are due to the trapped electrons oscillating longitudinally in the ponderomotive wave,[2-7] while the betatron oscillations are predominantly transverse electron oscillations due to the transverse spatial gradients associated with the wiggler field.[8-14] If the radiation wavefronts are curved these two oscillations can be resonantly coupled and can lead to electron detrapping in the ponderomotive wave.[1,15-18] When the radiation wavefronts are curved the electrons which are undergoing transverse betatron oscillations will experience periodically different phases in the ponderomotive wave. When the periodically changing phase, due to the transverse betatron oscillations, is resonant with the electron's synchrotron oscillations these later oscillations can be amplified and result in detrapping.

A very qualitative model for this process can be seen by considering the pendulum equation[1-3,6-7,15-18] for electrons in a wiggler field with transverse spatial gradients and a radiation field with a curved wavefront. For electrons deeply trapped in the ponderomotive wave the pendulum equation, roughly speaking, reduces to a driven harmonic oscillator equation. The characteristic frequency of the electron oscillation is the sychrotron frequency and is proportional to the fourth root of the radiation field power. The amplitude of the periodic driving term in the oscillator equation is proportional to the inverse of the radius of curvature associated with the radiation field and the period of the driving term is proportional to the electron betatron period. The electron phase, described by the pendulum

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equation, can be amplified if the frequency of the driving term is resonant with the characteristic trapping frequency. This detrapping mechanism could limit the level of radiation power generated by the FEL.

In this paper we suggest and analyze an alternative mechanism for sychrotron-betatron resonant detrapping which does not depend on the curvature of the radiation wavefronts. We show that even for a one dimensional radiation field sychrotron-betatron resonant detrapping can take place for tapered wiggler fields, i.e., the wiggler magnetic field amplitude.

To analyze this synchrotron-betatron resonant detrapping mechanism we choose a tapered linearly polarized wiggler field described by the vector potential

$$\underset{\sim}{A}_w(y,z) = A_w(z)\cosh\left(k_w(z)y\right)\cos\left(\int_o^z k_w(z')dz'\right)\hat{e}_x, \qquad (1)$$

where $A_w(z)$ and $k_w(z)$ are the spatially slowly varying amplitude and wavenumber. It will be assumed later that $k_w(z)y$ is somewhat less than unity so that $\cosh(k_w y) \approx 1 + (k_w y)^2/2$. The one dimensional radiation field is described by the vector potential

$$\underset{\sim}{A}_L(z,t) = A_L \sin(kz-\omega t+\theta)\hat{e}_x, \qquad (2)$$

where the amplitude A_L, wavenumber $k = \omega/c$, frequency ω and phase θ are assumed constant. Since (1) and (2) are independent of the x coordinate, the electron's canonical momentum in the x direction is conserved, i.e., $d(\underset{\sim}{P} - |e|(\underset{\sim}{A}_w + \underset{\sim}{A}_L)/c) \cdot \hat{e}_x/dt = 0$ where $\underset{\sim}{P} = \gamma m_o \underset{\sim}{v} = (P_x,P_y,P_z)$ denotes the electron's mechanical momenta. The relativistic particle orbit equations become

$$\dot{P}_y = \frac{-|e|^2}{2\gamma m_o c^2}\,\frac{\partial A^2(y,z,t)}{\partial y}\ , \qquad (3a)$$

$$\dot{P}_z = \frac{-|e|^2}{2\gamma m_o c^2} \frac{\partial A^2(y,z,t)}{\partial z} , \qquad (3b)$$

where $\underset{\sim}{A}(y,z,t) = \left(\underset{\sim}{A}_w(y,z) + \underset{\sim}{A}_L(z,t)\right) \cdot \hat{e}_x$ is the total x component of the vector potential and $\gamma = (1 + \underset{\sim}{P} \cdot \underset{\sim}{P}/m_o^2 c^2)^{1/2}$. In obtaining (3) we used the fact that

$$P_x = \frac{|e|}{c} A(y,z,t) + P_{ox}, \qquad (4)$$

and assumed that the injected momentum in the x direction is zero, i.e., $P_{ox} = 0$. Using (3) and (4) the electron's axial velocity, v_z, is given by

$$\dot{v}_z = \frac{-|e|^2}{2\gamma^2 m_o^2 c^2} \left(\frac{\partial}{\partial z} + \frac{v_z}{c^2} \frac{\partial}{\partial t}\right) A^2(y,z,t). \qquad (5)$$

In what follows it proves convenient to perform a transformation from the independent time variable t to the independent position variable z. An electron's phase with respect to the ponderomotive wave, in terms of the position variable z, is defined as

$$\tilde{\psi}(y_o,\psi_o,\psi_o',z) = \psi_o + \int_o^z \left(k_w(z') + k - \omega/\tilde{v}_z(y_o,\psi_o,\psi_o',z')\right)dz', \qquad (6)$$

where y_o is the electron's y position at $z = 0$, ψ_o is the phase at $z = 0$, $\psi_o' = \partial\tilde{\psi}/\partial z\big|_{z=0} = k_w(0) + k - \omega/v_o$ and v_o is the axial velocity at $z = 0$ (assumed to be identical for all electrons). Quantities denoted with the superscript ~ are functions of the initial condition variables y_o,ψ_o,ψ_o' and the independent variable z. Thus, for example, $\tilde{v}_z$ is the axial velocity at position z of an electron with initial condition (at z = 0) variables

y_o, ψ_o and ψ_o'. Differentiating (6) twice yields

$$\tilde{\psi}'' = k_w' + \omega \tilde{v}_z' / \tilde{v}_z^2, \tag{7}$$

where $'$ denotes the operator $\partial/\partial z$. Upon performing the operation $\partial/\partial z + v_z c^2 \partial/\partial t$ in (5), transforming the resulting equation from the independent variable t to the variable z and substituting the result into (7), we arrive at the generalized pendulum equation for a tapered wiggler with transverse spatial gradients,

$$\begin{aligned}\tilde{\psi}'' = k_w' - \frac{|e|^2 \omega}{2\tilde{\gamma}^2 m_o^2 c^2 \tilde{v}_z^3} & \Big[\big(2A_w A_w' \cosh^2(k_w \tilde{y}) + A_w^2 k_w' \tilde{y} \sinh(2k_w \tilde{y}) \big) \cos^2\big(\int_o^z k_w dz' \big) \\ & - k_w A_w^2 \cosh^2(k_w \tilde{y}) \sin\big(2 \int_o^z k_w dz' \big) \\ & + \big(A_w' A_L \cosh(k_w \tilde{y}) + A_w A_L k_w' \tilde{y} \sinh(k_w \tilde{y}) \big) \sin\tilde{\psi} \\ & + (k_w + k - \tilde{v}_z \omega / c^2) A_w A_L \cosh(k_w \tilde{y}) \cos\tilde{\psi} \Big] \end{aligned} \tag{8}$$

where $\tilde{v}_z = \omega/(k_w + k - \tilde{\psi}')$ and $\tilde{y} = \tilde{y}(z)$. The last term in (8) represents the usual ponderomotive potential wave.

Before going on to simplify (8) an equation describing the electrons betatron motion (transverse oscillations in y) is needed. Using (3a) and assuming $|k_w y| << 1$ and $|A_L| << |A_w|$ we find that

$$\ddot{y} = \frac{-|e|^2 A_w^2 k_w^2}{\gamma^2 m_o^2 c^4} \cos^2\left(\int_o^z k_w(z')dz'\right)y. \tag{9}$$

In arriving at (9) we have also assumed that $|\dot{\gamma}\dot{y}| \ll \gamma\ddot{y}$, these approximations can be shown to be well satisfied. Transforming (9) from the independent variable t to z and setting $\tilde{v}_z = c$ yields the following equation for the betatron orbits,

$$\tilde{y}'' + k_\beta^2(z)\left(1 + \cos\left(2 \int k_w(z')dz'\right)\right)\tilde{y} = 0, \tag{10}$$

where $k_\beta(z) = |e|A_w k_w/(\sqrt{2}\ \tilde{\gamma} m_o c^2) = \beta_w k_w/\sqrt{2}$ and $\beta_w = |e|A_w(z)/\tilde{\gamma} m_o c^2$ is the normalized wiggle velocity. Since the betatron wavenumber, k_β, is much greater than k_w, the betatron motion can be separated into a slowly varying part and a small rapidly varying part. Neglecting the rapid variations in $\tilde{y}(z)$, the solution of (10) is

$$\tilde{y}(z) = \tilde{y}(0)\left(k_\beta(0)/k_\beta(z)\right)^{1/2} \cos\left(\int_o^z k_\beta(z')dz' + \tilde{\phi}(0)\right), \tag{11}$$

where $\tilde{y}(0)$ and $\tilde{\phi}(0)$ are constants.

We can now proceed to simplify the pendulum equation in (8). Assuming $(k_w\tilde{y})^2 \ll 1$, $A_w' \ll k_w A_w$, $k_w' \ll k_w^2$ and keeping only the slowly varying terms, (8) reduces to the form

$$\frac{d^2\tilde{\psi}}{dz^2} + K_s^2(z)\cos\tilde{\psi} = \frac{dk_w}{dz} - \alpha^2\left[\frac{dA_w^2}{dz}\right.$$

$$\left. + \xi(z)\left(1 + \cos\left(2 \int_o^z k_\beta dz' + 2\tilde{\phi}(0)\right)\right)\right], \tag{12}$$

where $K_s(z) = \left(|e|^2 \omega k_w A_w A_L / (\tilde{\gamma}^2 m_o^2 c^5)\right)^{1/2}$ is the synchrotron wavenumber, $\alpha^2 = |e|^2 \omega / (4\tilde{\gamma}^2 m_o^2 c^5)$, $\xi(z) = \tilde{y}^2(0)\left(k_\beta(0)/2k_\beta(z)\right) d(A_w k_w)^2/dz$ and (11) was used to replace $\tilde{y}(z)$. We can further simplify (12) by noting that

$$\left| \alpha^2 \xi(z) \right| << \frac{dk_w}{dz},\ \alpha^2 \frac{dA_w^2}{dz} .$$

Therefore we may keep only the driving term associated with the betatron oscillations which could amplify the synchrotron oscillations, i.e., $\cos\left(2 \int_o^z k_\beta dz' + 2\tilde{\phi}(0)\right)$, and (12) reduces to

$$\frac{d^2\psi}{dz^2} + K_s^2(z)\sin\psi = \frac{dk_w}{dz}$$

$$- \alpha^2 \left[\frac{dA_w^2}{dz} + \xi(z)\cos(2 \int_o^z k_\beta dz')\right], \tag{13}$$

where for later convenience we have shifted the phase $\tilde{\psi}$ by $\pi/2$, i.e. $\psi = \tilde{\psi} + \pi/2$, and set $\tilde{\phi}(0) = 0$. Note that the electron's energy, neglecting transverse wiggler gradients, is determined by the relation

$$\partial\tilde{\gamma}/\partial z = - \frac{|e|^2 A_w A_L k}{2\tilde{\gamma} m_o^2 c^4} \sin\psi. \tag{14}$$

Defining the resonant phase, ψ_R, in the usual way, i.e.,

$$\sin\psi_R = \left(dk_w/dz - \alpha^2 dA_w^2/dz\right)/K_s^2, \tag{15}$$

the pendulum equation (13) becomes

$$\frac{d^2\psi}{dz^2} + K_s^2 \sin\psi = K_s^2\left(1 + \varepsilon \cos(2 \int_o^z k_\beta dz')\right)\sin\psi_R$$

$$- \varepsilon\left(1 + \frac{\beta_w^2}{2} \frac{k}{k_w}\right) \frac{dk_w}{dz} \cos(2\int_o^z k_\beta dz'), \qquad (16)$$

where $\varepsilon = k_w^2 \tilde{y}^2(0)/2$.

We now consider the particularly simple illustration of a wiggler field with a constant period ($\partial k_w/\partial z = 0$) and linearly changing amplitude, $A_w(z) = A_w(0) + \delta A_w z/L_w$, where $A_w(0) >> |\delta A_w|$ is constant and L_w is the length of the wiggler field. The pendulum equation in (13) for the case where $|\delta A_w/A_w(0)| << 1$, $K_s = 2k_\beta/(1 + \delta)$ and $|\delta| << 1$, i.e., the synchrotron and betatron oscillations are resonant, reduces to

$$\frac{d^2\psi}{dZ^2} + \sin\psi = \left(1 + \varepsilon \cos(1 + \delta)Z\right)\sin\psi_R, \qquad (17)$$

where $Z = K_s z$ and $\sin\psi_R = - \gamma_z^2\left(\delta A_w/A_w(0)\right)/(2k_w L_w)$. For the case where A_w is constant and $k_w(z) = k_w(0) + \delta k_w z/L_w$ varies linearly, the pendulum equation in (13) reduces to

$$\frac{d^2\psi}{dZ^2} + \sin\psi = \left(1 - \varepsilon\beta_w^2\gamma_z^2 \cos(1 + \delta)Z\right)\sin\psi_R, \qquad (18)$$

where $\sin\psi_R = (\delta k_w/k_w(0)/\left(2\beta_w^2 L_w k_w(0)\right)$.

Rather than obtain an approximate solution to (17) or (18), using a multiple time scale approach, we simply solve (17) numerically. Initially the phases are distributed uniformly between 0 and 2π with $\partial\tilde{\psi}/\partial z = 0$. Figure (1) shows the precentage of trapped particles as a function of normalized distance for $\sin\psi_R = 0.3$ and $\varepsilon = 0.1$ and 0.15. Initially approximately 65% of the particles are trapped and for $\varepsilon = 0$ this fraction is of cause maintained. After about 5 synchrotron oscillations 55% are trapped for $\varepsilon = 0.1$ and 50% for $\varepsilon = 0.15$. Since we have assumed zero beam emittance, these results apply only to those

electrons initially on the outer edge of the beam, i.e., those having the largest value of ε. Figure (2) shows the percentage of particles detrapped after 10 synchrotron oscillations as a function of the mismatch parameter δ, for $\sin\psi_R = 0.3$ and $\varepsilon = 0.1, 0.15$. The percentage of detrapped particles maximize when $\delta < 0$ since the more deeply trapped particles oscillate slightly faster than those trapped nearer the phase space sepratrix. Here again these results apply to only a small fraction of the total number of beam electrons, those initially near the edge of the beam.

Our model is somewhat idealized since, among other things, beam emittance has been neglected. The neglect of emittance implies that all the electrons have the same initial betatron phase, $\tilde{\Phi}(0)$, see Eq. (11). Hence, those electrons injected near the axis will not experience the betatron synchrotron detrapping since the value of ε for these electrons is much smaller than those near the edge of the beam.

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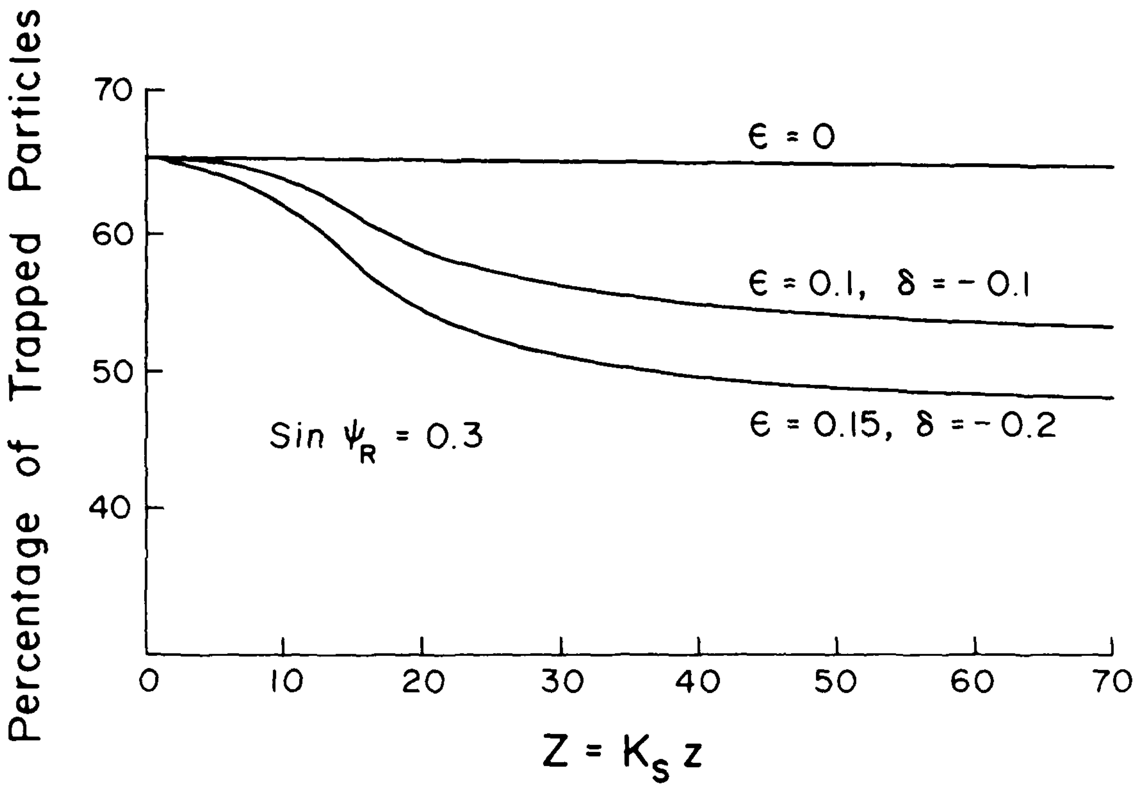

Fig. 1 — Percentage of trapped electrons as a function of normalized distance for $\sin\psi_R = 0.3$, and two values of ϵ.

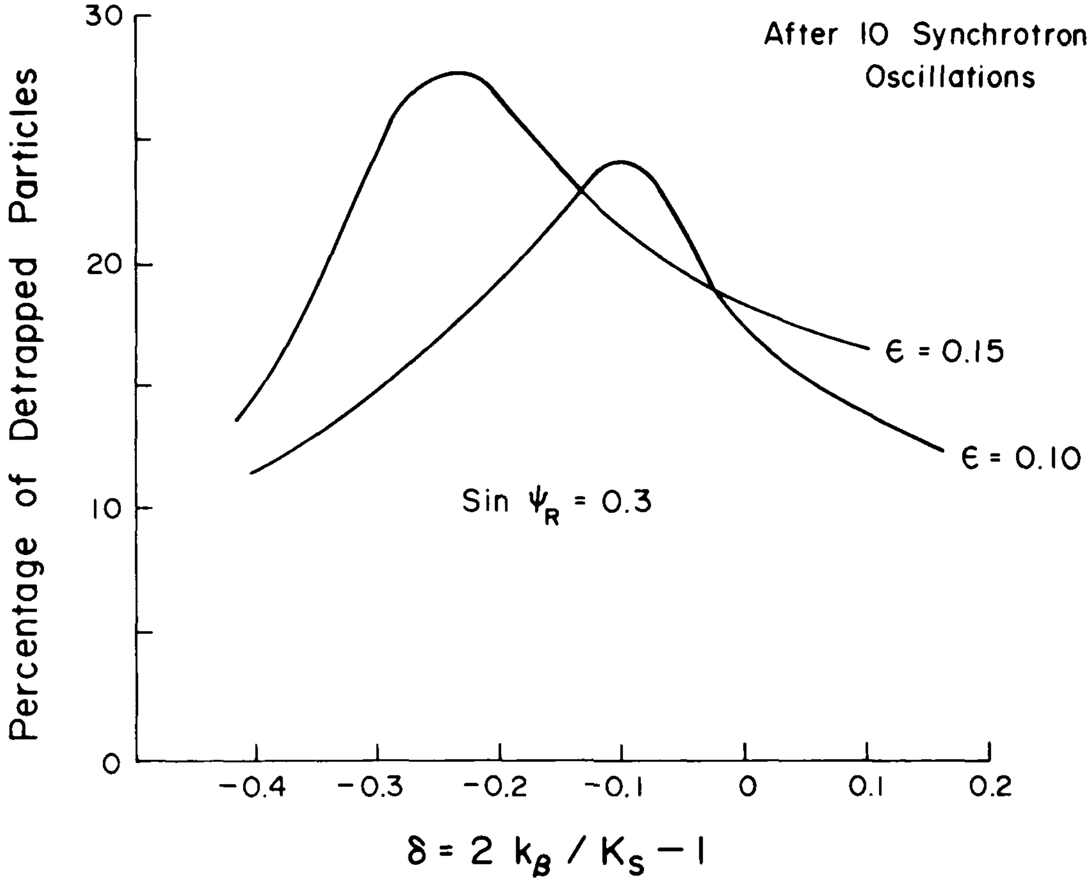

Fig. 2 — Percentage of trapped electrons after 10 synchrotron oscillations as a function of the mismatch parameter δ for $\sin\psi_R = 0.3$ and $\epsilon = 0.1$, and 0.15.

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